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# LOW ENERGY THEOREMS OF BROKEN SCALE INVARIANCE FOR LIGHT SCALAR MESONS

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Low energy theorems of broken scale invariance are discussed from the point of view of $1/N_c$ expansion. A simple effective Lagrangian model realizing these theorems and predicting scalar gluonium as well as quarkonium masses is presented.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Низкоэнергетические теоремы нарушенной масштабной инвариантности для легких скалярных мезонов

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С точки зрения $1/N_c$ разложения обсуждаются низкоэнергетические теоремы нарушенной масштабной инвариантности. Предлагается простая модель эффективного лагранжиана, выполняющего эти теоремы, и предсказываются массы скалярных глюония и кваркония.

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While at the classical level QCD possesses invariance when quark masses are neglected, this symmetry is broken by quantum effects [1] expressed in the anomalous trace $(\theta_\mu{}^\mu)_{an}$ of the energy-momentum tensor $\theta_{\mu\nu}$ generating non-conservation of the dilatation current $\mathfrak{D}_\mu$, i.e.,

$$\partial_\mu \mathfrak{D}^\mu = (\theta_\mu{}^\mu)_{an} = -\frac{b}{8}\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{\mu\nu}_a , \qquad (1)$$

where $G^a_{\mu\nu}$ are the gluonic field strength tensors, $b = (11/3)N_c - (2/3)N_F$ with $N_c$ and $N_F$ being the numbers of colours and flavours, respectively. As a consequence, one can derive the following low energy theorem [2]

$$F(0) = \frac{b}{2} G_0 \qquad (2)$$

for the two-point function

$$F(q^2) = i \int d^4x\, e^{iqx} \langle 0|T(H(x)\, H(0))\,|0\rangle \qquad (3)$$

of the scalar gluonic current $H \equiv -(\theta_\mu{}^\mu)_{an}$, $G_0 = \langle 0|(\alpha_s/\pi) G^a_{\mu\nu} G^{\mu\nu}_a\,|0\rangle$ is the gluon condensate.

In gludynamics with $N_c = 3$ and $N_F = 0$ eq. (2) combined with the assumption that (3) is dominated by a single scalar gluonium $\sigma$ gives a clear prediction concerning the gluonium mass $m_\sigma$ as follows:

$$f_\sigma^2 m_\sigma^2 = \frac{11}{2} G_0 \ , \qquad (4)$$

where $f_\sigma$ is an analogue of the pion decay constant $f_\pi$:

$$\langle 0|H|\sigma\rangle = m_\sigma^2 f_\sigma \ . \qquad (5)$$

Unfortunately we do not know $f_\sigma$ reliably to predict $m_\sigma$ from (4). Instead one can use (4) to determine the important parameter $f_\sigma$ as a function/3/ of $G_0$ and $m_\sigma$ with $m_\sigma$ estimated, e.g., by lattice calculations and/or by experiments. However, when massless quarks are included, i.e. $N_F \neq 0$, then also quarkonium states are created and one can ask whether (and in what sense) eqs.(2) and/or (4) could still be reliably used to predict masses (or other parameters) of the scalar particles (including quarkonia). We shall answer this quastion affirmatively within the framework of the $1/N_c$ expansion/4/.

To do this, let us remember that from the point of view of the $1/N_c$ expansion /4/ eq.(3) can be decomposed as follows

$$F(q^2) = F_0(q^2) + F_1(q^2) \ , \qquad (6)$$

where $F_0(q^2)$ is a sum of all planar diagrams without quark loops while $F_1(q^2)$ is the sum of all such diagrams with quark loop included. Thus, $F_0(q^2)$ is of the leading order $O(N_c^2)$ in $1/N_c$ while $F_1(q^2)$ is $O(N_c)$ since each quark loop is suppressed by a factor $1/N_c$. $F_0(q^2)$ knows nothing about quarks and the only singularities of $F_0(q^2)$ are one-gluonium poles, so it contains dominant information on glue states. Comparing the lowest order in $1/N_c$ terms on both sides of eq.(2) for a large-$N_c$ limit we get

$$F_0(0) = \frac{11N_c}{6} G_0 \ . \qquad (7)$$

This is the exact result of the $N_c \to \infty$ limit and for the real world with $N_c = 3$ eq.(7) must be taken only as a prediction of large $N_c$ dynamics. We shall see later that this prediction, in fact, leads to the pure gluodynamics result (4) as expected on general grounds of $1/N_c$ expansion /4/. Having this in mind we can combine prediction (7) with (2) and (6) and obtain

$$F_1(0) = -\frac{N_F}{3} G_0 \ . \qquad (8)$$

Since on the planar diagram level $F_1(q^2)$ includes quark loop contributions, then on a phenomenological level eq.(8) should be understood as a prediction concerning properties of scalar quarkonia (their masses, mixings with gluonia, etc.) while (7) concerns only gluonia.

We shall illustrate all this more explicitly within a simple effective Lagrangian model [5] containing a scalar gluonium field $\sigma(x)$, pion fields $\pi_i(x)$ (i = 1,2,3) and a flavour (u, d) singlet scalar quarkonium field $S(x)$, i.e., $N_F = 2$. The quark matter fields S and $\pi_i$ are assumed to form the genuine linear sigma model for chiral SU(2)xSU(2) symmetry while the flavourless gluonium field $\sigma$ is invariant under the chiral SU(2)xSU(2) transformations. The Lagrangian is of the form [5]

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}[(\partial_\mu S)^2 + (\partial_\mu \pi_i)^2] - V(\sigma, S, \pi_i), \qquad (9)$$

where the potential V is chiral invariant and obeys the trace anomaly equation

$$(\theta_\mu^{\ \mu})_{an} \equiv - H_0 \left(\frac{\sigma}{f_\sigma}\right)^4 = 4V - \sigma\frac{\partial V}{\partial \sigma} - S\frac{\partial V}{\partial S} - \pi_i \frac{\partial V}{\partial \pi_i}, \qquad (10)$$

where $H_0 = (b/8)G_0$. Eq.(10) guarantees [3] that the Ward identity (2) is realized in the present model. We neglect the quark mass term and assume that SU(2)xSU(2) symmetry is spontaneously broken. So, we shall use the following reparametrization of the fields $\sigma$ and S [6]:

$$\sigma(x) = f_\sigma \exp\left(\frac{\tilde{\sigma}(x)}{f_\sigma}\right), \qquad (11)$$

$$S(x) = f_s + \tilde{S}(x),$$

where $f_\sigma = \langle 0|\sigma|0\rangle$, $f_s = \langle 0|S|0\rangle$ and $\tilde{\sigma}, \tilde{S}$ and $\pi_i$ are the correct fields with the VEV's equal to zeros. The potential V is assumed not to contain derivatives of fields and it can only be a function of the two SU(2)xSU(2) invariants $\sigma$ and $(S^2 + \pi_i^2)^{1/2}$.

Solving eq.(10) we find

$$V(\sigma, S, \pi_i) = H_0 \left(\frac{\sigma}{f_\sigma}\right)^4 \ln\frac{\sigma}{C} + \sigma^4 f\left(\frac{\sqrt{S^2+\pi_i^2}}{\sigma}\right), \qquad (12)$$

where C is an arbitrary constant of the mass dimension and f is an arbitrary dimensionless function of the argument $(S^2 + \pi^2)^{1/2}/\sigma$. Since the gluonium field $\sigma$ is an SU(2)xSU(2) singlet, it does not change the form of the SU(2) axial current $A_\mu^i = S\partial_\mu \pi_i - \pi_i \partial_\mu S$, so we can use $\langle 0|A_\mu^i|\pi^j\rangle = i f_\pi p_\mu \delta^{ij}$ to deduce that [6]

$$f_S = -f_\pi , \tag{13}$$

where the pion decay constant $f_\pi = 93$ MeV. Eliminating terms linear in $\tilde{\sigma}$ and $\tilde{S}$ from potential (12) by requiring $\langle 0| \frac{\partial V}{\partial \tilde{\sigma}} |0\rangle = \langle 0| \frac{\partial V}{\partial \tilde{S}} |0\rangle = 0$ we find the following mass relations from (12) and (13) [5,6]:

$$m_\pi^2 = 0 ,$$

$$f_\sigma m_{\sigma s}^2 = 2 f_\pi m_{ss}^2 , \tag{14}$$

$$f_\sigma^2 m_{\sigma\sigma}^2 - f_\pi^2 m_{ss}^2 = \frac{b}{2} G_0 ,$$

where the $m_{ij}^2$ are entries in the squared mass matrix for the $\tilde{\sigma}$ and $\tilde{S}$ fields. For the two-point function (3) we get (in tree approximation):

$$F(q^2) = \frac{(f_\sigma m_{\sigma\sigma}^2 - \frac{1}{2} f_\pi m_{\sigma s}^2)^2 (m_{ss}^2 - q^2)}{(m_{\sigma\sigma}^2 - q^2)(m_{ss}^2 - q^2) - \frac{1}{4} m_{\sigma s}^4} . \tag{15}$$

We see from (14) and (15) that the present model realizes the Ward identity (2). Having in mind that [4] $m_{\sigma\sigma}^2$ and $m_{ss}^2$ are $O(N_c^0)$, $m_{\sigma s}^2 \sim O(1/\sqrt{N_c})$, $f_\sigma \sim O(N_c)$ and $f_\pi \sim O(\sqrt{N_c})$ we easily extract the leading order in $1/N_c$ term $F_0 \sim O(N_c^2)$ from (15) as follows

$$F_0(q^2) = \frac{f_\sigma^2 m_{\sigma\sigma}^4}{m_{\sigma\sigma}^2 - q^2} . \tag{16}$$

The term $F_1(q^2) \sim O(N_c)$ is then given simply as a difference of eqs. (15) and (16) (see (6)). Thus, for $N_c = 3$ and $N_F = 2$ eqs. (6)-(8) and (14)-(16) give the following formulae

$$f_\sigma^2 m_{\sigma\sigma}^2 = \frac{11}{2} G_0 \tag{17}$$

and

$$f_\pi^2 m_{ss}^2 = \frac{2}{3} G_0 . \tag{18}$$

We see that prediction (17) coincides with (4) found in gluodynamics as one expects on general grounds [4]. On the other hand, it is amusing to use (18) with $G_0 = 0.012$ GeV$^4$ [7] to predict $m_{ss} = 960$ MeV. This can be compared with the mass $m_\delta = 980$ MeV of a possible isovector scalar quarkonium $\delta(980)$ and we notice good agreement.

It is worth to mention here that although a decomposition like (6) takes place [8] also for analogous two-point function of the pseudoscalar gluonic current $\alpha_s G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$, one cannot go further in the analogy since unlikely to $0^+$ channel the inclusion of massless quarks in $0^-$ channel requires [8] the $(\text{mass})^2 \sim O(1/N_c)$ for the pseudoscalar flavour singlet quarkonium $\eta'$ in order to have cancellation of the contributions from $F_0$ and $F_1$ - like pieces. Such a behaviour of the pseudoscalar channel is needed [8] to remove the vacuum angle $\theta$ - dependence of the theory in the chiral limit. However, in $0^+$ channel no such a behaviour of $F(0)$ is demanded because the r.h.s. of eq.(2) is not zero for any value of $N_c$, large or small, and so scalar particle masses have expected [4] behaviour $O(N_c^0)$.

The results (17) and (18) can be nicely presented in terms of a special form $V_s$ of potential (12) if one makes a choice

$$f\left(\frac{\sqrt{S^2+\pi_i^2}}{\sigma}\right) = -\frac{1}{6}\frac{G_0}{f_\sigma^4}\ln\frac{C}{f_\pi}\frac{\sqrt{S^2+\pi_i^2}}{\sigma} + \frac{1}{24}\frac{G_0}{f_\pi^4}\left(\frac{\sqrt{S^2+\pi_i^2}}{\sigma}\right)^4 . \tag{19}$$

This translates (12) into

$$V_s(\sigma, S, \pi_i) = \frac{G_0}{24}\left(\frac{\sigma}{f_\sigma}\right)^4\left[11N_c\ln\frac{\sigma}{C} - 4\ln\frac{\sqrt{S^2+\pi_i^2}}{f_\pi}\right] + \frac{1}{24}\frac{G_0}{f_\pi^4}(S^2+\pi_i^2)^2 , \tag{20}$$

where $N_c = 3$. We have, however, written here the general dependence on $N_c$ explicitly in order to do evident that (20) is a sum of terms of increasing order in $1/N_c$. The logarithmic term in (19) is required in order to extract the pure gluodynamics potential from the rest of eq.(12). This potential is the first (leading order in $1/N_c$) term in (20) and gives (17) if $4\ln(f_\sigma/C) = -1$ to eliminate the linear $\tilde{\sigma}$ - dependence of $V_s$. On the other hand, to eliminate the linear $\tilde{S}$-dependence from $V_s$, the second term in (19) is demanded unambiguously. Thus, potential (20) is a minimal one dictated by the large-$N_c$ limit of QCD giving (17) and (18) (see also ref. [9]). If we neglect fluctuations of the gluonium field, i.e., we put $\sigma = f_\sigma$ in (20), we obtain just a potential $V_{LSM}$ for the linear sigma model in the chiral limit as follows

$$V_{LSM}(S, \pi_i) = -\frac{G_0}{6}\ln\frac{\sqrt{S^2+\pi_i^2}}{f_\pi} + \frac{1}{24}\frac{G_0}{f_\pi^4}(S^2+\pi_i^2)^2 , \tag{21}$$

where we neglet unimportant constant term. The potential (21) together with corresponding mass relation (18) have recently been derived by Andrianov et al.[10] directly from QCD in their procedure of bosonization.

We conclude that the bosonization of ref.[10] does not yet include gluonia, and the only remnant of gluonic degrees of freedom that remains in the approach of [10] is the appearance of gluonic condensate $G_0$ giving a non-zero mass to an isosinglet scalar $(u, d)$ quarkonium $S$. Inclusion of gluonia should lead to more general potential (20) (or even (12)) realizing the total QCD Ward identity (2) while (21) realizes only a piece of (2), namely (8). Thus, it seems to us that the result of ref.[10] could encourage efforts to derive an effective Lagrangian realizing (2) by including gluonia as well as quarkonia directly from QCD by integrating over the quark and gluon fields[11].

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